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ON THE BEHAVIOR AND DESIGN OF THIN-WALLED COLUMNS HAVING DOUBLY SYMMETRIC SECTIONS

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INTRODUCTION

The principal cause of failure of thin-walled columns is the nonlinear mode interaction of local and Euler-type buckling. The local buckling undermines the stiffness of the structure and, thus, hastens the overall buckling of the column. For smaller ratios of overall to local buckling stresses ($\alpha<1$), the columns are highly imperfection-sensitive and failure is by snap-through initiated in the elastic range of material behavior. For larger values of α, the failure is precipitated by plastic yielding of the material caused by the combined effects of local and overall deformations. In either case, the failure occurs at a load which is often considerably smaller than the Euler buckling load of the column.

The literature on the subject of interactive buckling of columns is vast; but, it is only relatively recently that the essential features of the phenomenon have been clearly understood. In a previous paper (10), a new analytical model has been developed for a study of interactive buckling in thin-walled beam-columns with doubly symmetric sections. This model has shed some new light on the phenomenon and thus offers itself as a tool for a critical appraisal of the current design philosophy of thin-walled columns. In the present paper, some of the prominent features of the behavior of thin-walled columns subject to interactive buckling are illustrated, taking typical examples. This is followed by a comparison of the theoretical collapse loads as given by the new model with the test results currently available in the field. A design approach based on the effective width concept (5) is then selected because of its intuitive appeal and simplicity for an examination of its performance. Potential areas of weakness of this and a similar approach given by the new AISI code (9) are indicated. A statistically derived knock-down factor is suggested as an interim design aid to avoid unsafe designs.

THE NEW ANALYTICAL MODEL

Reference (10) gives an outline of the analytical model mentioned above. The model is primarily developed for doubly symmetric sections, but can be readily extended to singly symmetric sections. The theory will not be repeated here, and the interested reader is referred to (1) and (10). The essential features of this model will now be briefly recapitulated to facilitate an appreciation of the key points of this paper:

1. The interaction of overall bending/buckling of the column with the local buckling mode associated with the lowest critical stress (σ_1) - henceforth called the "primary local mode" - triggers a secondary local buckling mode having the same wavelength as the primary one. In a realistic interactive buckling analysis of a doubly symmetric compression member, these "companion" modes must be considered together. If the primary mode is symmetric (antisymmetric) with respect to the axis of bending, then the secondary mode is antisymmetric (symmetric) with respect to the same axis. This feature was demonstrated in Ref. (10) and (11). Thus, the new analytical model incorporates the two companion local modes. These are illustrated in Fig. 1(a-d) for I-section and box columns respectively.

2. The amplitudes of the two local modes do not remain constant, but must be given freedom to vary along the length of the member. This phenomenon of "amplitude modulation" is demonstrated in Ref. (10) and is accounted for in the model in an elegant manner using the concept of a "slowly varying" function first introduced by Koiter (7).

3. The model employs a new beam element which has cubic polynomial shape functions for the description of overall axial and lateral displacements and linear variations for the functions modulating the local amplitudes. The cross-sectional deformations as given by the local modes together with the associated second order fields are duly embedded into the element. The description of overall displacements in terms of a F.E. model makes it possible to deal with arbitrarily prescribed end-conditions and to account for changes in the overall displacement profile which occur due to interaction with the local modes under continued loading. However, this paper deals exclusively with columns simply supported at both the ends.

The rigorous treatment of the secondary local mode in the new model has demonstrated that imperfection-sensitivity for higher values of is much smaller than predicted by earlier asymptotic theories and the severity of mode interaction increases as the ratio (γ) of the secondary local critical stress (σ_2) to σ_1 approaches unity (i.e. $\gamma \rightarrow 1$) (11).

As the locally buckled columns bend under the influence of the overall imperfections, the eccentricity of axial load or the presence of lateral loads, the amplitude of the local modes vary along the length of the column depending upon the magnitude and sign of the bending moment developed in the column. This results in the accentuation of the edge stresses of the plate elements in the buckles located in the region of maximum bending moment. Thus, plastic yielding occurs at a much smaller average stress than would be predicted by a theory which does not consider amplitude modulation.

ESSENTIAL FEATURES OF THE PHENOMENON

In this section, some of the essential features of the phenomenon of interactive buckling of columns with doubly symmetric sections will be briefly outlined. For simplicity, the columns would be assumed to have small imperfections in the primary local and the overall mode.

1. The column bends in the overall sense and develops local buckling deformation in the primary mode under axial compression from the very beginning of loading history. It is interesting to note that although no imperfections are present in the secondary local mode, displacements in this mode occur under the combined action of the primary local and overall modes. In physical terms, the local buckling deformations are aggravated on the compression side of the bending axis of the section and alleviated on the tension side.

2. The closer the value of α to unity, the more severe the imperfection-sensitivity, i.e. the reduction of the maximum load carrying capacity of the column. This has been known for a long time (6) and has been the theme of several publications, e.g. (2,7,8,13). This is seen from a comparison of load-end-shortening characteristics of I-section (ξ_0) in Fig. 3(a). Again, the smaller the imperfection magnitudes, the more rapid would be the unloading after reaching the peak load under displacement-controlled loading [Fig. 3(a)].

3. The failure of elastic thin-walled columns under displacement-controlled loading can be in either of the following modes:

   (i) The column would reach a peak load followed by an unloading phase. This would be generally the case, if the values of α lies close to unity. Figure 3(a) illustrates such a behavior for I-section columns with dimensions indicated in Fig. 2(a) with values of α=1.06 and 1.65, respectively. The same behavior is exhibited by the square box column [α=1.02, cross-section as in Fig. 2(b)], as shown in Fig. 3(b).

   (ii) The column would develop huge deflections as the load approaches an asymptotic value. Such a behavior is typical of columns with α>>1, e.g. the I-section column [cross-section as in Fig. 2(a)] with α=2.9, and the square box column [cross-section as in Fig. 2(b)] with α=1.8, whose load-end shortening characteristics are shown in Fig. 3(a) and (b), respectively. This behavior resembles that of an Euler column with a reduced critical load.

It is interesting to note that the I-section column in Fig. 2(c) exhibits behavior indicated in (i) for a value of $\alpha \simeq 2.0$, whereas the lipped I-section column in Fig. 2(d) with α=1.01 exhibits the behavior in (ii) (not illustrated). This indicates that the effects of mode interaction are less severe for members built up of stiffened elements with low slendernesses than for those composed predominantly unstiffened plate elements with high slendernesses.

4. As already pointed out, the amplitudes of the local modes (ξ_1 and ξ_2) vary along the length of the column. The variation in the amplitude of the secondary local model (ξ_2) is more significant than that of the primary mode in the range of stresses less than σ_2, for then, it occurs primarily by an interaction of bending moment (which varies along the length of the column) with the primary local mode. Typical variations of ξ_1 and ξ_2 are shown in Fig. 4 for the case of a square box column with α=1.8, corresponding to the point A in Fig. 3(b).

5. In practical steel columns, plastic yielding would often intervene and cut short the load carrying capacity of the column. Apparently, this is the only way columns exhibiting the behavior indicated in 3.(ii) above, will fail. In thin members, plastic yielding precipitates some form of localized plastic buckling or corner collapse. Thus, it is unrealistic to depend on any inelastic reserve capacity in interactive buckling, the load corresponding to the first yield being a good approximation to the load carrying capacity of the column. As mentioned eariler, an analysis which does not take into account the phenomenon of amplitude modulation can err, often by a significant margin in the prediction of the first yield.

Comparison with the Currently Available Results

The results given by the new model have been checked against the currently available theoretical results on stiffened panels and square box columns given by Koiter. The results on stiffened panels are "exact" and the agreement in this case is extremely close (12). The results on square box columns were obtained using a number of approximations, but in the range of their applicability (α<1.3), the agreement is still satisfactory (10,11). A comparison with a test on square box column is also presented elsewhere (10).

Test Results from Sydney and Cornell Universities

The experimental results from the University of Sydney, Australia and Cornell University, USA, constitute valuable data for a critical appraisal of the theoretical model (4,5). Though both of the series of tests were conducted on thin-walled I-sections, there are some significant differences in the geometric and material properties of the specimens tested. Briefly, these are:

(i) Sydney columns were fabricated by welding the web with the flanges, and these are of the same thickness. This produces weld residual stresses which were measured. The Cornell columns, on the other hand, were fabricated by glueing channels back to back. This gives rise to wide differences in the values of γ of the two sets of I-sections.

(ii) Thickness of the sheet steel used for fabrication of Sydney specimens was about four times as much as that of Cornell specimens. Also, the slendernesses of the flange outstands were considerably higher for the Cornell columns.

(iii) The initial imperfections in the forms of local distortions and overall bowing were measured in Sydney tests, even though they proved to be extremely small. Imperfections were not measured for Cornell columns; but, because of the slenderness of the plate elements involved, it is probably reasonable to assume relatively higher local distortions as well as overall imperfections.

(iv) All the Sydney columns carried loads which were off the center by known amounts and some tests were designed to have the load eccentricity of about 1/1000 of the length of the column.

Failure Criterion in the Present Study

Unless the column collapsed in the elastic range - which is signalled by a limit point on the equilibrium path, the maximum carrying capacity of the column was deemed to have been attained at the initiation of plastic yielding. Stresses are examined at the middle surface of the plate elements, any surface yield due to the bending of plate elements being ignored.

Comparison with Sydney Results

Reference 4 gives the full details of the test, including averaged distribution of residual stresses across the column sections, the initial imperfections and measured eccentricities of applied load. The averaged residual stress distributions were used in the determination the local buckling stresses, and the agreement with the reported experimental results were very close. These will not be discussed here. Maximum out of the plane plate imperfection was observed to be 0.03t to 0.04t (t = the plate thickness) the actual component in the mode of buckling remaining unknown. In the present study, an averaged uniform imperfection in the sense of the primary mode of amplitude 0.01t (ξ_1=0.01) was assumed. Though maximum overall imperfection was reported to be L/5000 (L = length of the column), it is not known whether these were in the sense of cancelling with or additive to the effect of end eccentricities of axial load. In order to reduce the effects of this uncertainty, the measured eccentricities alone were used in the calculations, with ξ_0 (the overall imperfection amplitude divided by flange thickness) being set to zero. The residual stresses, though measured, show considerable scatter, and this could seriously blur the prediction of first yield at a critical section. This difficulty was summarily dealt with by simply stepping down the yield stress of the material by an averaged value of the compressive residual stress σ_r given by

$$\sigma_r = \sigma_1{}^* - \sigma_1{}^r \tag{1}$$

where $\sigma_1{}^*$ and $\sigma_1{}^r$ are the local critical stresses of the column completely free of residual stresses and carrying the nominal averaged pattern of residual stresses, respectively.

Table 1 shows a comparison of the theoretical results of σ_u/E (σ_u = average collapse stress, E = Young's modulus of the material) as given by the new theory and the experimental results. It is seen the agreement is quite close in all cases except specimens #3, 7, and 13. Specimen #3 is a case of near-coincident buckling and is far more sensitive to initial imperfections than the other two. Assuming an overall imperfection of L/5000 in all cases, the theoretical values of σ_u/E are found to be within 6%, 10%, and 15% respectively of the corresponding experimental values of #3, 13, and 7, respectively. Considering the reported variation of residual stress distribution, sheet thickness, and the difficulties of maintaining a truly hinged end condition, the agreement between theory and experiment appears to be very satisfactory indeed.

Comparison with Cornell University Tests

Reference 5 documents the full details of test on I-section columns tested. The sheet steel used for fabrication of specimens is about 0.05 in (1.25mm) thick. Reference 5 suggests that the maximum out of plane deflection of the plate elements was of the order of 0.2t. It is reasonable to suppose that the imperfection amplitude in the sense of the primary local mode was smaller than this value. The overall imperfections under laboratory conditions tend to be of the order of L/5000, though small-scale columns of the type tested at Cornell can have a higher level of imperfection, say L/3000.

Table 2 shows the comparison of theoretical and experimental results of σ_u/E for two typical levels of initial imperfections, viz.,

(i) $\bar{\xi}_0$ = L/3000t and $\bar{\xi}_1$ = 0.20

and (ii) $\bar{\xi}_0$ = L/5000t and $\bar{\xi}_2$ = 0.05

It is found that most of the experimental results lie in between or quite close to one of the theoretical values with some exceptions of shorter columns (viz. LCI-1, LCII-1, and LCIII-1). Apparently assuming the overall imperfection as a given fraction or the length has resulted in an underestimate of the overall imperfections in these cases. Assuming an overall imperfection magnitude equal to thickness of the sheet (1.25mm $\simeq$ L/1000) brings the theoretical values to within a range of 5% of the experimental results at least for the columns LCII-1 and LCIII-1. Thus, the theory is seen to be capable of predicting the collapse loads of thin- walled columns very satisfactorily indeed. Table 2 illustrates the crucial role of imperfections.

CORNELL UNIVERSITY DESIGN APPROACH: AN APPRAISAL

A simple analysis based on effective width concept in conjunction with an appropriate column curve was used by Kalyanaraman et al. (5) to predict the collapse strength of their test specimens. A variation of this approach is recommended as a design tool in the current draft specifications of the AISI. The approach does not take into account the influence of the secondary local mode and the imperfection-sensitivity of columns with α less than or in the close vicinity of 1. Notwithstanding, the author believes that the technique is a useful one and, further, it can be improved to yield accurate values of the collapse strength of columns with practical levels of imperfections.

The main parameters controlling the behavior of thin-walled columns may be taken to be: σ_0/E, σ_1/E, σ_2/E, σ_y/E, $\bar{\xi}_0$ and $\bar{\xi}_1$ or, alternatively, α, β $(=\sigma_0/\sigma_y)$, γ, δ $(=\sigma_y/\sigma_1)$, $\bar{\xi}_0$ and $\bar{\xi}_1$. Of these, we may select $\bar{\xi}_0$ and $\bar{\xi}_1$, keeping in view the practical difficulties of fabrication and erection and the thin-walled nature of the structure. Also, σ_0 can be readily calculated. σ_1/E and σ_2/E remain and require to be accurately calculated for a realistic prediction of the ultimate strength of the column. Even though such a calculation is considered difficult at present, several efficient computer routines based on the finite strip procedure are currently available and it is expected that such values will become available as standard properties of the section.

Currently, the effective widths are calculated taking k=4.0 (in the relationship $\sigma_1/E = k[\pi^2/12(1-\nu^2)][t/w]^2)$, w being the width of the plate element for stiffened elements, and a value of 0.43 is recommended for unstiffened elements in the draft AISI specifications. The latter value is unduly conservative, and the former value is not necessarily safe. The use of the correct value of the local critical stress in the effective width formula would more truly reflect the interaction of plate elements in the local buckling process. Thus, in the present study, the following formulae for effective width (b_e) are used:

$$\frac{b_e}{w} = \eta(1-0.22\eta) \tag{2a}$$

for stiffened elements; and

$$\frac{b_e}{w} = 1.19\eta(1-0.297\eta) \tag{2b}$$

for unstiffened elements. In these formulae, $\eta=\sqrt{\sigma_1/\sigma_e}$, σ_e is the edge stress assumed to be uniform over the effective width, and w is the actual width of the plate element. Note that σ_1 is the correct value of the minimum primary local critical stress determined by a rigorous analysis taking various integer values for the number of half-waves of buckling. The use of the correct value of σ_1 does not, however, remove the limitations of the method, which were mentioned earlier. It also becomes apparent that the reported "good" agreement between the theory and experiment in Ref. 5 is merely a fortuitous result of a significant underestimate of the local critical stresses by the approximate procedure used therein (3).

Table 3 gives the details of including the key parameters of a selection of columns investigated. Table 4 gives the results of σ_u/E obtained by the "improved Cornell" approach and the present theory. No residual stresses are considered in the calculations. A scrutiny of the table yields the following conclusions:

(i) Cornell University approach is clearly unconservative for column with $\alpha\leq1$. This is particularly true of I-columns built placing channels back to back, as then the values of γ are much smaller ($\simeq1.2$) than when the flanges and the web are of the same thickness ($\gamma\simeq2.8$). Similar observation holds good when there is an interaction of imperfections and plastic yielding as when $\delta<1.2$(say). It appears that this effect is controlled by the product $\alpha\gamma\delta$.

(ii) The predictions are too conservative for columns with high values of β. This is probably associated with a significant underestimate of the average effective stress in the range $\sigma_e<\sigma_y/2$ by the CRC column formula used in the approach. For a given ratio of overall imperfection to the length of the column, the rate of growth of overall deflections becomes smaller as β increases - a feature which is not taken into account in the CRC column formula. This effect can be masked by a low value of γ.

(iii) The approach gives more satisfactory results for columns consisting of stiffened elements than for ones consisting predominantly of unstiffened elements.

The Provisions of the New AISI Specifications

The latest (1986) specifications of AISI (9) recommend a new effective width formula in terms of a slenderness factor which depends on k, the buckling coefficient. With 'k' taken as 4.0 and 0.43 respectively for the stiffened and unstiffened elements, the formula is applicable for both types of element. The effective width of each element is then calculated at a stress F_u given by

$$F_u = \sigma_u \qquad \text{if } F_u < \sigma_y/2$$

and

$$F_u = \sigma_y(1-\sigma_y/4\sigma_0) \qquad \text{if } F_u > \sigma_y/2$$

The ultimate load P_u of the column is given by

$$P_u = A_e F_u$$

where A_e is the effective area calculated at F_u. The great advantage of the procedure is that no iterations are involved. At the time this paper is being written, only a limited number of calculations have been made for purposes of comparison, and the results are shown as $\sigma_u{}^\alpha/E$ in Table 4. It is interesting to note that the results of the "improved" Cornell approach and the new AISI specifications do not differ a great deal. As a matter of fact, the new AISI Specifications yield more accurate results in a few cases, e.g. LCV-3 (#1) (I-section - t_w=2t) and (190-4800, I-section - t_w=t). But the sensitivity to imperfections are still not accounted for, and, thus, the method can produce designs which are unsafe for values of α less than or in the neighborhood of unity.

A Correction Factor

On the basis of theoretical studies using the new model, it appears that a simple empirical formula for the prediction of the collapse load of thin-walled columns can be developed. This formula would involve the predicted collapse load as given by the improved Cornell University approach (which uses the exact critical stresses in the expressions for the effective widths) multiplied by a factor which is dependent on α, β, γ, and δ. The effect of initial imperfections of ξ_0=1/1000 and ξ_1=0.1 have been built into the formula. After a number of trials, it appears that the most satisfactory form of the formula is:

$$\sigma_u = \lambda \sigma_u{}^c,$$

in which

$$\lambda = C \left(1+\alpha\gamma\delta\right)^m \left(1+ \frac{1}{\alpha\gamma\delta}\right)^n \left(1+\gamma\right)^p \left(1+ \frac{1}{[1+\beta]^2}\right)^q$$

where m, n, p, q and C are constants calculated for the best fit with the available results, and $\sigma_u{}^c$ is the prediction of the collapse stress as given by the "improved" Cornell University approach. For columns consisting mainly of unstiffened elements (typically I-section columns), the values of the constants are:

$m = -0.0312$, $n = -0.1120$, $p = 0.3789$,

$q = -0.5344$, and $C = 0.708$.

For columns built up mainly of stiffened elements (e.g., box columns), the values of the constants are:

$m = 0.1888$, $n = 0.3964$, $p = 0.2250$,

$q = -0.6011$, and $C = 0.600$.

These formulae are designed to give not more than 5% error on the unconservative side; but, an error of about 10% is sometimes possible on the safe side.

CONCLUSION

Features of the phenomenon of interactive buckling of thin-walled columns are briefly reviewed. The theoretical results are compared with the recent test results from University of Sydney and earlier results from Cornell University. The agreement is very good indeed. The role of imperfections in producing scatter in the prediction of the ultimate capacity of the columns is illustrated. Design procedures, one developed at Cornell University in the seventies and the other recommended by the new AISI Code, are examined in the light of the new results. In order to account for the interaction of plate elements, it is suggested that exact local critical stress be used in the effective width formulae. To obtain reliable results for σ_u/E which will allow for some unavoidable imperfections in the column and other limitations inherent in the Cornell approach, it is proposed that the collapse load obtained from the "improved" Cornell University approach be multiplied by a factor. This factor must be a function of the key parameters governing the problem. A statistically derived expression for this factor is also suggested.

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APPENDIX II. NOTATION

The most important symbols are defined below:

E	Young's Modulus
L	Length of the column
a	Halfwave length of local buckling
b	The shorter side of the box column
c	The longer side of the box column
r	The radius of gyration of cross-section
t	Thickness of flange of I-section columns and of a side in box columns
t_w	Thickness of web
$\alpha, \beta, \gamma, \delta$	Key parameters, viz. σ_0/σ_1, σ_0/σ_y, σ_2/σ_1 and σ_1/σ_y, respectively.
σ	The average axial stress carried by the column
σ_u	The maximum average axial stress carried by the column
σ_y	The yield stress of the material
ξ_0, ξ_1, ξ_2	The nondimensional amplitudes of overall and local buckling (the maximum deflection in the cross-sectional profile divided by t)
$\bar{\xi}_0, \bar{\xi}_1, \bar{\xi}_2$	The nondimensional imperfections in the modes of buckling

Table 1. Comparison Between Theory and Experiment: Sydney Tests

NO.	DESIGNATION	L/t	e/t	$\frac{\sigma_1}{E}\times10^3$	$\frac{\sigma_2}{E}\times10^3$	$\frac{\sigma_0}{E}\times10^3$	EXPER. $\frac{\sigma_u}{E}\times10^3$	THEORY $\frac{\sigma_u}{E}\times10^3$
1	190-2000A	480.4	0.04	1.157	3.943	3.299	1.39	1.38
2	190-2000B	480.4	0.53	1.157	3.943	3.299	1.21	1.28
3	190-3400A	754.9	0.04	1.164	3.993	1.338	1.04	1.18
4	190-3400B	754.9	0.75	1.164	3.993	1.338	0.85	0.91
5	190-4800B	990.6	1.08	1.286	4.389	0.719	0.62	0.56
6	240-2800A	613.2	0.08	0.805	2.870	2.996	1.29	1.27
7	240-2800B	637.3	0.39	0.721	2.628	2.996	1.04	1.21
8	240-4200A	911.7	0.14	0.721	2.600	1.461	0.99	0.95
9	240-4200B	911.7	0.73	0.721	2.600	1.461	0.81	0.85
10	240-5800A	1179.2	0.22	0.806	2.731	0.810	0.79	0.73
11	240-5800B	1179.2	1.32	0.806	2.731	0.810	0.67	0.62
12	310-3600A	794.1	0.20	0.423	1.589	3.204	1.10	1.13
13	310-3600B	794.1	0.59	0.423	1.589	3.204	0.92	1.04
14	310-5800A	1179.3	0.08	0.480	1.712	1.347	0.74	0.76
15	310-5800B	1225.5	0.99	0.430	1.574	1.347	0.61	0.67

Table 2. Comparison between Theory and Experiment: Cornell Tests*

NO.	COLUMN DESIG-NATION	$\frac{\sigma_1}{E} \times 10^3$	$\frac{\sigma_2}{E} \times 10^3$	$\frac{\sigma_0}{E} \times 10^3$	$\sigma_u \times 10^3$ (Exper-iment)	$\sigma_u/E \times 10^3$ (Theory)$^+$ (i)	(ii)
1	LCI-1	0.292	0.342	3.851	0.495	0.621	0.662
2	LCI-2	0.294	0.348	1.851	0.484	0.485	0.499
3	LCI-3	0.288	0.339	1.051	0.390	0.372	0.392
4	LCII-1	0.389	0.451	3.391	0.610	0.670	0.692
5	LCII-2	0.378	0.449	1.392	0.492	0.466	0.491
6	LCII-3	0.379	0.449	0.755	0.372	0.351	0.379
7	LCIII-1	0.542	0.660	2.789	0.668	0.742	0.770
8	LCIII-2	0.516	0.622	0.992	0.466	0.472	0.520
9	LCIII-3	0.524	0.632	0.504	0.366	0.325	0.382
10	LCIV-1	0.744	0.858	2.018	0.658	0.712	0.778
11	LCIV-2	0.780	0.941	1.011	0.572	0.582	0.673
12	LCIV-3	0.795	0.946	0.615	0.472	0.433	0.521
13	LCV-1	1.081	1.258	1.752	0.842	0.840	0.964
14	LCV-2	1.071	1.254	1.133	0.702	0.683	0.825
15	LCV-3	1.085	1.268	0.639	0.620	0.483	0.574

* Note the imperfection levels for the predictions (i) and (ii) are given in the text.

$^+$ σ_y/E varies slightly from specimen to specimen with a mean value of about 1.02×10^{-3}.

Table 3. Identification and Key Parameters of the Columns Investigated

NO.	TYPE OF COLUMN	α	β	γ	δ	$\frac{\sigma_y}{E} \times 10^3$
	I-SECTION (Ref. 5) (Table 2)					
1	LCV-3	0.59	0.58	1.17	1.01	1.10
2	LCIV-3	0.77	0.59	1.19	1.31	1.04
3	LCIII-3	0.95	0.59	1.20	1.64	0.86
	I-SECTION (Ref. 4) (Table 1)					
4	190-2000	2.85	1.66	2.72	1.18	1.99
5	190-4800	0.56	0.36	2.78	1.10	1.99
	SQUARE BOX COLUMNS (Fig. 2b)					
6	(L=2640t)	0.84	0.51	1.44	1.65	1.67
7	(L=2640t)	0.84	0.85	1.44	0.99	1.00
8	(L=1200t)	4.04	2.45	1.44	1.65	1.67
9	(L=1200t)	4.04	4.08	1.44	0.99	1.00
	RECTANGULAR BOX COLUMNS (c/b=2, b/t=60)					
10	(L=5000t)	0.85	0.17	1.14	5.15	1.67
11	(L=5000t)	0.85	0.28	1.14	3.09	1.00
12	(L=2000t)	5.33	1.04	1.14	5.14	1.67
13	(L=2000t)	5.33	1.73	1.14	3.09	1.00
	I-SECTION WITH LIPPED FLANGES (Fig. 2d)					
14	(L=1833t)	0.82	0.58	1.02	1.40	2.67
15	(L=940t)	3.10	2.20	1.02	1.40	2.67

Table 4. Comparison of the Cornell Approach and the Present Theory

NO.	TYPE OF COLUMN	$\frac{\sigma_u^C}{E} \times 10^3$	$\frac{\sigma_u^T}{E} \times 10^3$	ERROR %	$\frac{\sigma_u^a}{E} \times 10^3$
	I-SECTION (Ref. 5) (Table 2)				
1	LCV-3	0.57	0.44	+23*	0.53
2	LCIV-3	0.49	0.39	+25*	0.47
3	LCIII-3	0.36	0.29	+24*	0.38
	I-SECTION (Ref. 4) (Table 1)				
4	190-2000	1.31	1.59	-18	1.25
5	190-4800	0.70	0.62	+13*	0.67
	SQUARE BOX COLUMNS (Fig. 2b)				
6	(L=2640t)	0.72	0.66	+9*	0.70
7	(L=2640t)	0.63	0.61	+3*	0.62
8	(L=1200t)	1.01	1.20	-16	1.01
9	(L=1200t)	0.75	0.86	-10	0.75
	RECTANGULAR BOX COLUMNS				
10	(L=5000t)	0.23	0.19	+21*	0.23
11	(L=5000t)	0.23	0.19	+21*	0.23
12	(L=2000t)	0.58	0.78	-26	0.64
13	(L=2000t)	0.46	0.60	-23	0.51
	I-SECTION WITH LIPPED FLANGES				
14	(L=1833t)	1.33	1.21	+10*	1.29
15	(L=940t)	1.74	2.15	-19	1.71

* Error on the unsafe side.

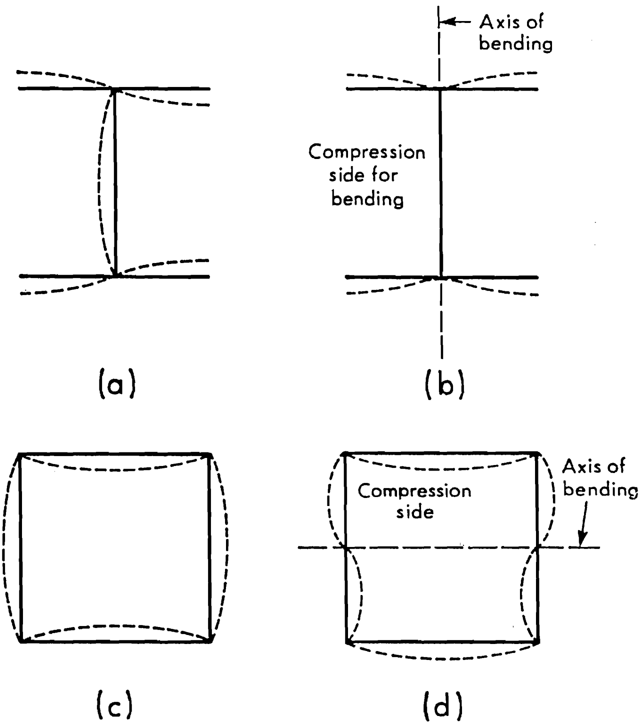

Figure 1(a-d). Typical primary (a and c) and secondary local modes (b and d).

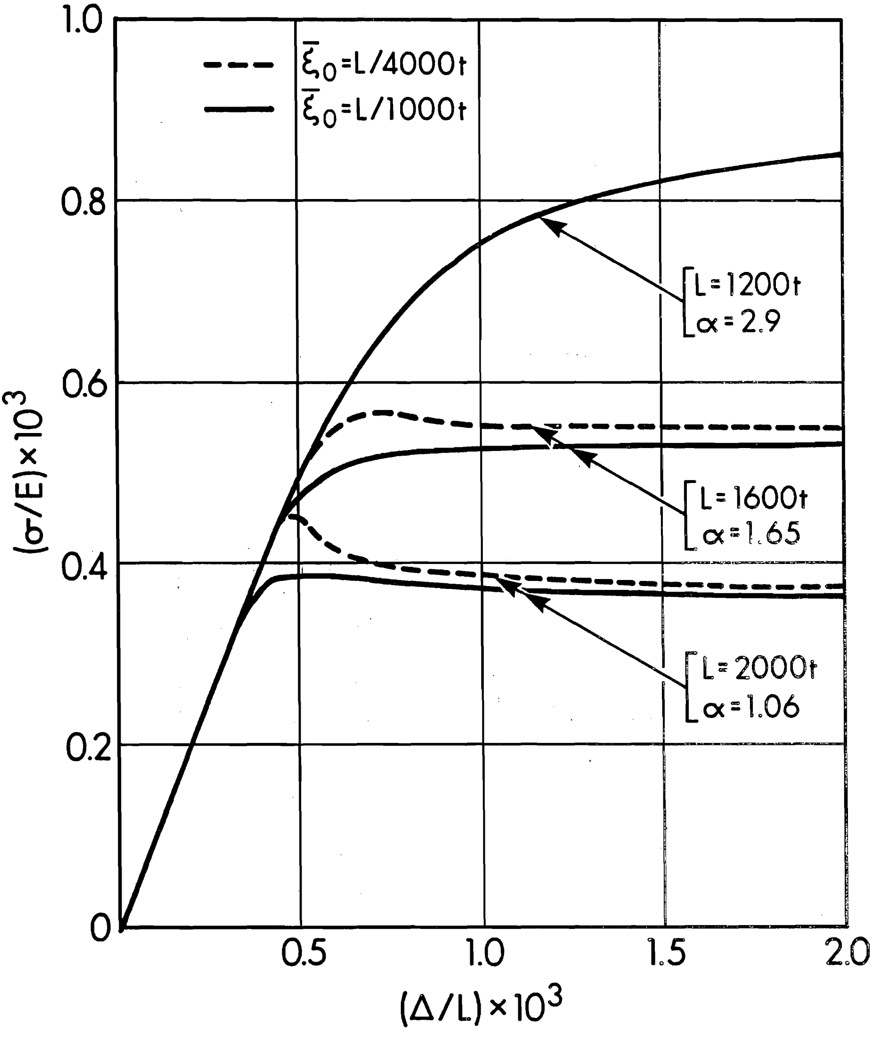

Figure 3(a). Average stress (σ) vs. end-shortening (Δ) for the I-section (Fig. 2(a)) ($\bar{\xi}_1$=0.1)

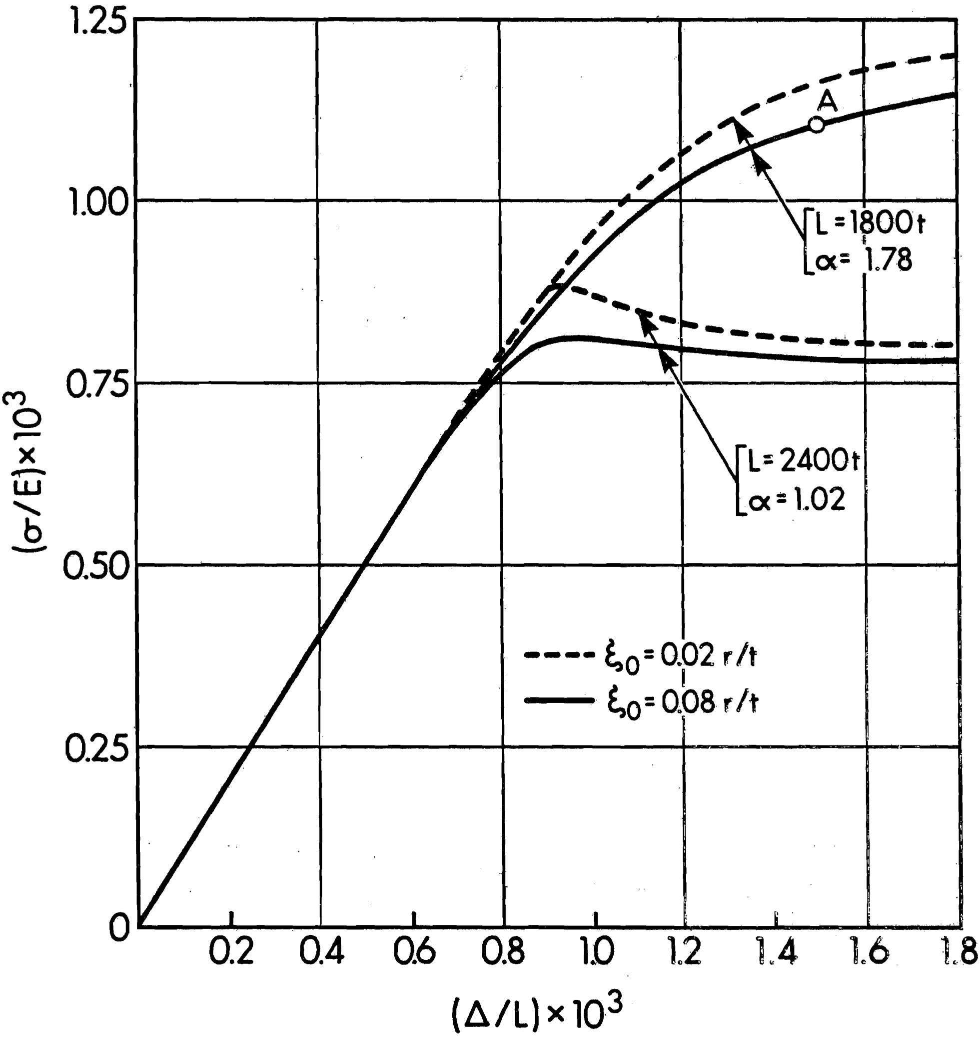

Figure 3(b). Average stress (σ) vs. end-shortening (Δ) for a square box column (Fig. 2(b)) ($\bar{\xi}_1 = 0.025$; $\bar{\xi}_0$ is given in terms of r, the radius of gyration of the section)

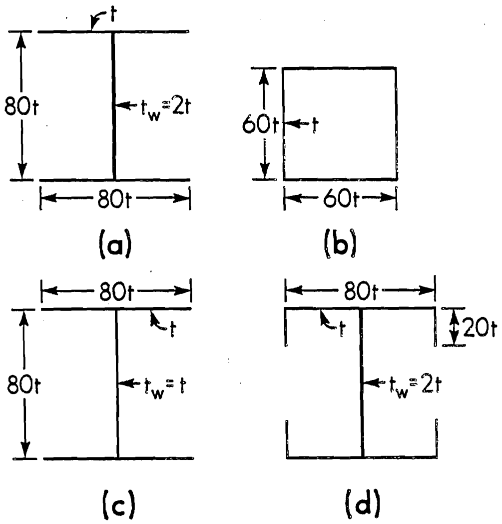

Figure 2. Cross-sections of columns investigated.

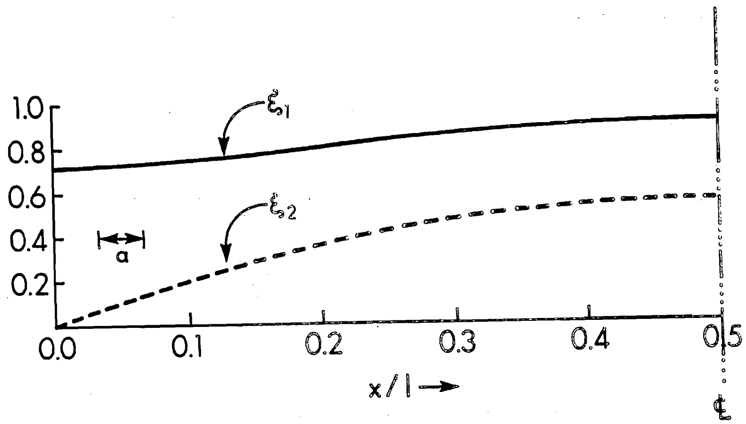

Figure 4. The variation of nondimensional amplitudes (the deflection contribution at the middle of the compression flange divided by "t") along the length of the column.